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云辐射参数化研究

张峰

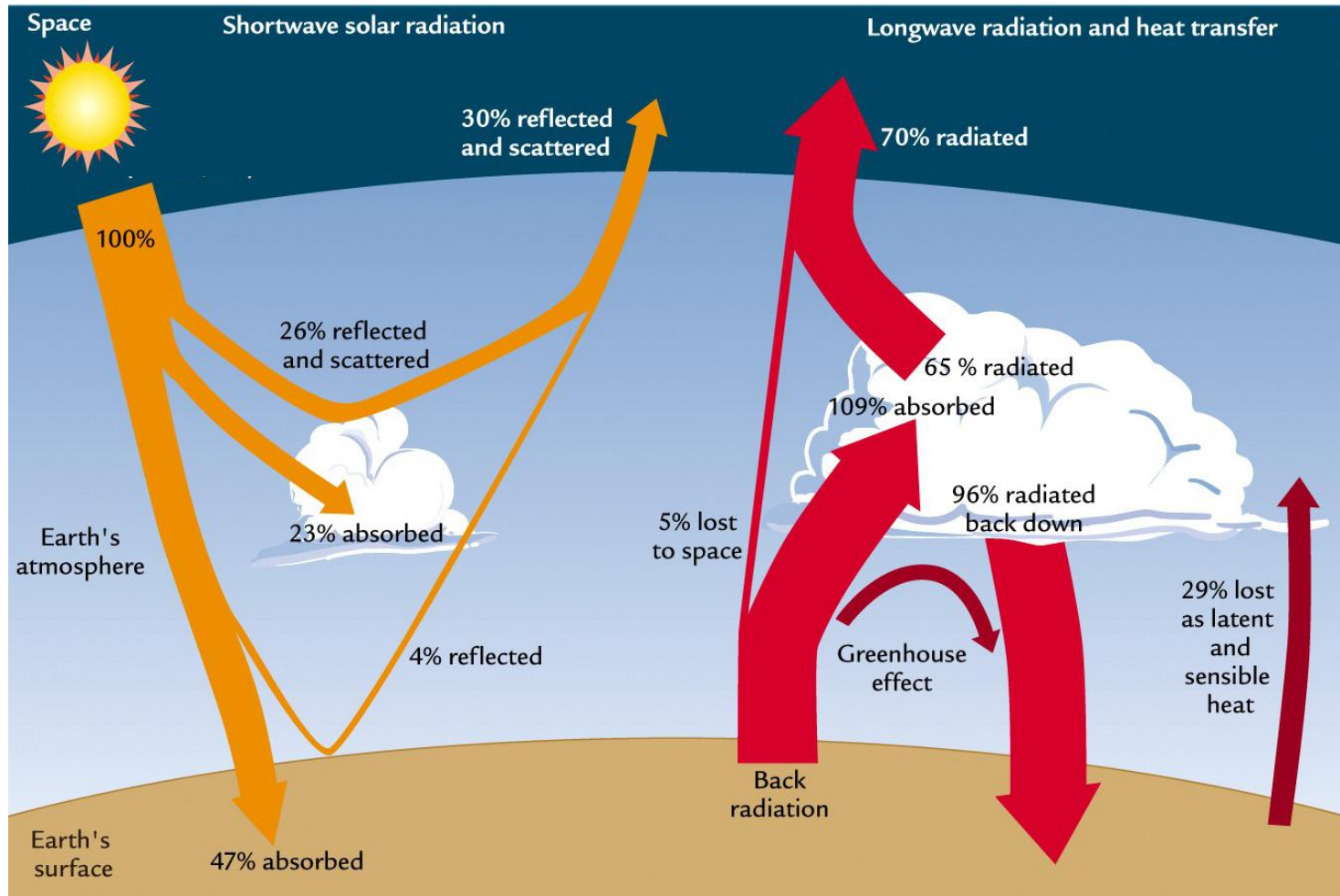
复旦大学 大气与海洋科学系/ 大气科学研究院

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Introduction

Cloud Radiative transfer (RT) process is one of the basic physical processes of the earth-atmosphere system, which plays an important role in the climate change and remote sensing.

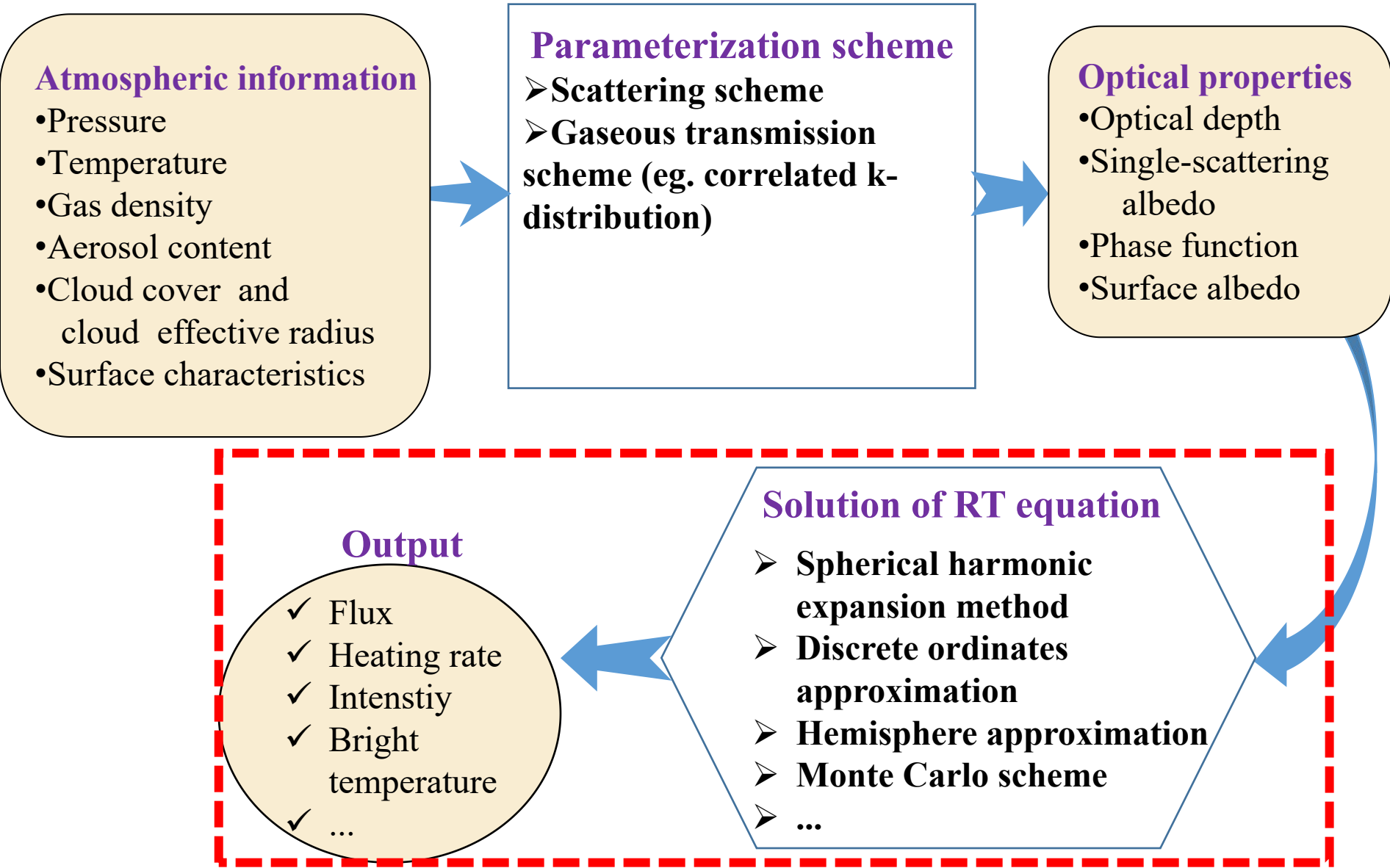


- 1. Cloud radiative transfer for solar radiation**
- 2. Cloud radiative transfer for Infrared radiation**
- 3. Radiative transfer in vertically internally inhomogeneous cloud**
- 4. Radiative transfer model (ERTM) and its application**



Introduction

Framework of Radiative transfer simulation





Cloud radiative transfer for solar radiation

Introduction

The radiative transfer equation is an integro-differential equation. The exact solution of the radiative transfer equation in a scattering and absorbing media is impossible to be obtained in a computationally efficient manner even for the plane-parallel case, thus, approximate methods are necessary.

Solar radiative transfer equation

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - \frac{\omega}{2} \int_{-1}^1 I(\tau, \mu') P(\mu, \mu') d\mu' - \frac{\omega}{4\pi} F_0 e^{-\tau/\mu_0} P(\mu, -\mu_0)$$

single-scattering
albedo

cosine of
zenith angle

radiative
intensity

azimuthal in-
dependent phase
function

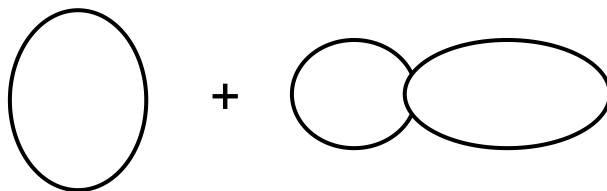


Cloud radiative transfer for solar radiation

The δ -two-stream approximations provide a simple answer to radiative transfer, especially the δ -Eddington approximation and δ -two-stream discrete ordinates method (DOM), are widely used.

δ -Eddington approximation

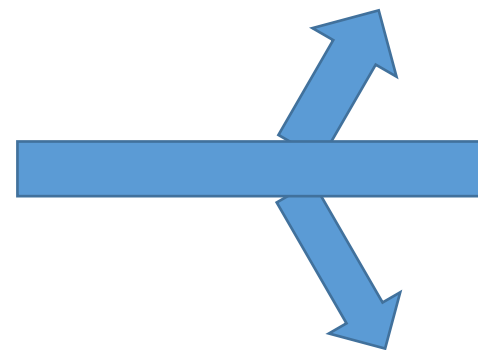
$$I(\tau, \mu) = I_0(\tau) + I_1(\tau)\mu$$



δ -two-stream discrete ordinates method (DOM)

$$\int_{-1}^1 F(\mu) d\mu = \sum_{i=-n}^n w_i F(\mu_i)$$

$$I(\tau, \mu) = I_{-1}(\tau, \mu_{-1}) + I_{+1}(\tau, \mu_{+1})$$



However, the cloud heating might have been underestimated by as much as 10% under the cloudy-sky condition, which indicates that a four-stream or higher order approximation scheme is necessary in order to obtain the accurate solar cloud absorption in weather and climate models.



Cloud radiative transfer for solar radiation

1. Adding method of four stream DOM (4DDA)

Single layer solution for four stream discrete ordinate method (DOM)
(Liou et al.1988)

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - \frac{\omega}{2} \int_{-1}^1 I(\tau, \mu') P(\mu, \mu') d\mu' - \frac{\omega}{4\pi} F_0 e^{-\tau/\mu_0} P(\mu, -\mu_0)$$

$$\mu_i \frac{dI_i}{d\tau} = I_i - \frac{\omega}{2} \sum_{l=0}^3 \omega_l P_l(\mu_i) \sum_{j=-2}^2 a_j I_j P_l(\mu_j) - \frac{\omega}{4\pi} F_0 \sum_{l=0}^3 \omega_l P_l(\mu_i) P_l(-\mu_0) e^{-\tau/\mu_0}, i = \pm 1, \pm 2$$



solution

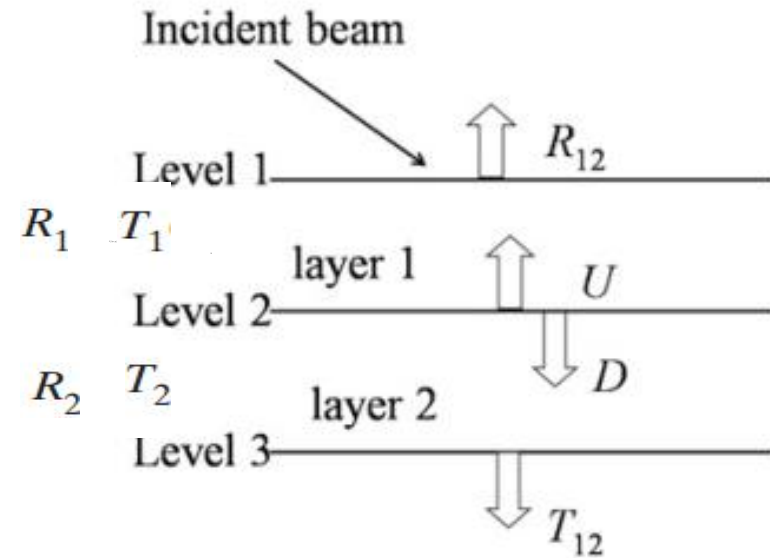
$$\begin{bmatrix} I_2 \\ I_1 \\ I_{-1} \\ I_{-2} \end{bmatrix} = \begin{bmatrix} \varphi_2^+ e^{-k_2 \tau} & \varphi_1^+ e^{-k_1 \tau} & \varphi_1^- e^{-k_1(\tau_0 - \tau)} & \varphi_2^- e^{-k_2(\tau_0 - \tau)} \\ \Phi_2^+ e^{-k_2 \tau} & \Phi_1^+ e^{-k_1 \tau} & \Phi_1^- e^{-k_1(\tau_0 - \tau)} & \Phi_2^- e^{-k_2(\tau_0 - \tau)} \\ \Phi_2^- e^{-k_2 \tau} & \Phi_1^- e^{-k_1 \tau} & \Phi_1^+ e^{-k_1(\tau_0 - \tau)} & \Phi_2^+ e^{-k_2(\tau_0 - \tau)} \\ \varphi_2^- e^{-k_2 \tau} & \varphi_1^- e^{-k_1 \tau} & \varphi_1^+ e^{-k_1(\tau_0 - \tau)} & \varphi_2^+ e^{-k_2(\tau_0 - \tau)} \end{bmatrix} \mathbf{G} + \begin{bmatrix} Z_2^+ \\ Z_1^+ \\ Z_1^- \\ Z_{-2}^- \end{bmatrix} e^{-f_0 \tau}$$

Cloud radiative transfer for solar radiation

Adding method of Four stream DOM (4DDA):

a) two layers

The four principles of invariance governing the reflection and transmission of light beam,



$$U(\mu, \mu_0) = R_2(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + 2 \int_0^1 R_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$

$$D(\mu, \mu_0) = T_1(\mu, \mu_0) + 2 \int_0^1 R_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$R_{1,2}(\mu, \mu_0) = R_1(\mu, \mu_0) + U(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu}\right) + 2 \int_0^1 T_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$T_{1,2}(\mu, \mu_0) = T_2(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + D(\mu, \mu_0) \exp\left(-\frac{\tau_2}{\mu}\right) + 2 \int_0^1 T_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$



Cloud radiative transfer for solar radiation

Using the two node Gaussian quadrature to decompose the integrations

$$\mathbf{U}(\mu_0) = \mathbf{R}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{R}}_2 \mathbf{D}(\mu_0)$$

$$\mathbf{D}(\mu_0) = \mathbf{T}_1(\mu_0) + \overline{\mathcal{R}}_1^* \mathbf{U}(\mu_0)$$

$$\mathbf{R}_{1,2}(\mu_0) = \mathbf{R}_1(\mu_0) + \overline{\mathcal{T}}_1^* \mathbf{U}(\mu_0)$$

$$\mathbf{T}_{1,2}(\mu_0) = \mathbf{T}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{T}}_2 \mathbf{D}(\mu_0)$$

We get the solution

$$\mathbf{U}(\mu_0) = [\mathbf{E} - \overline{\mathcal{R}}_2 \overline{\mathcal{R}}_1^*]^{-1} \left[\mathbf{R}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{R}}_2 \mathbf{T}_1(\mu_0) \right]$$

$$\mathbf{D}(\mu_0) = \mathbf{T}_1(\mu_0) + \overline{\mathcal{R}}_1^* [\mathbf{E} - \overline{\mathcal{R}}_2 \overline{\mathcal{R}}_1^*]^{-1} \times \left[\mathbf{R}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{R}}_2 \mathbf{T}_1(\mu_0) \right]$$

$$\mathbf{R}_{1,2}(\mu_0) = \mathbf{R}_1(\mu_0) + \overline{\mathcal{T}}_1^* [\mathbf{E} - \overline{\mathcal{R}}_2 \overline{\mathcal{R}}_1^*]^{-1} \times \left[\mathbf{R}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{R}}_2 \mathbf{T}_1(\mu_0) \right]$$

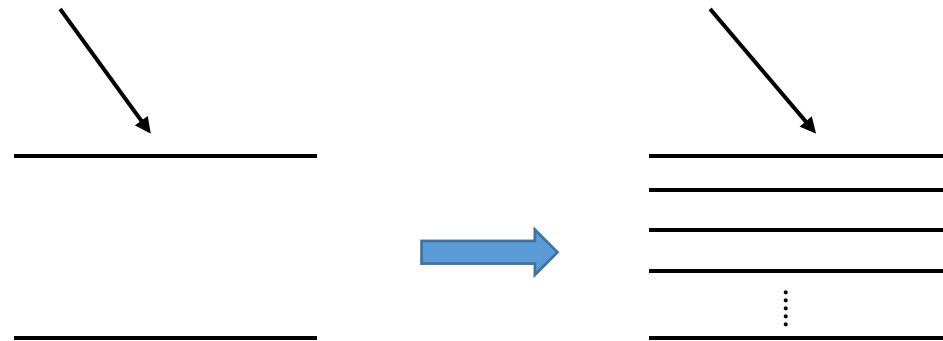
$$\mathbf{T}_{1,2}(\mu_0) = \mathbf{T}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{T}}_2 \mathbf{T}_1(\mu_0) + \overline{\mathcal{T}}_2 \overline{\mathcal{R}}_1^* [\mathbf{E} - \overline{\mathcal{R}}_2 \overline{\mathcal{R}}_1^*]^{-1}$$

$$\times \left[\mathbf{R}_2(\mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + \overline{\mathcal{R}}_2 \mathbf{T}_1(\mu_0) \right]$$



Cloud radiative transfer for solar radiation

b) multi-layer



Applied to an atmospheric slab extending from the layer 1 to layer k,

$$\mathbf{U}_{k+1}(\mu_0) = [\mathbf{E} - \bar{\mathbf{R}}_{k+1,N} \bar{\mathbf{R}}_{1,k}^*]^{-1} [\mathbf{R}_{k+1,N}(\mu_0) e^{-\tau_{1,k}/\mu_0} + \bar{\mathbf{R}}_{k+1,N} \mathbf{T}_{1,k}(\mu_0)]$$

$$\mathbf{D}_{k+1}(\mu_0) = \mathbf{T}_{1,k}(\mu_0) + \bar{\mathbf{R}}_{1,k}^* [\mathbf{E} - \bar{\mathbf{R}}_{k+1,N} \bar{\mathbf{R}}_{1,k}^*]^{-1} [\mathbf{R}_{k+1,N}(\mu_0) e^{-\tau_{1,k}/\mu_0} + \bar{\mathbf{R}}_{k+1,N} \mathbf{T}_{1,k}(\mu_0)]$$

Finally, the upward and downward fluxes at level k + 1 (lower boundary of the layer k) are

$$F_{k+1}^{\uparrow} = \mu_0 F_0 \boldsymbol{\mu} \cdot \mathbf{U}_{k+1}(\mu_0) \quad F_{k+1}^{\downarrow} = \mu_0 F_0 \boldsymbol{\mu} \cdot \mathbf{D}_{k+1}(\mu_0) + \mu_0 F_0 e^{-\tau_{1,k}/\mu_0}$$

Zhang et al J. Atmos. Sci., 2013a
Zhang et al J. Atmos. Sci., 2016



Cloud radiative transfer for solar radiation

2. Adding method of four SHM (4SDA)

Single layer solution for four stream spherical harmonic expansion method (SHM) (Li et al.1996)

The intensity can be separated out the angle-dependent factor by assuming

$$I(\tau, \mu) = \sum_{l=0}^3 I_l(\tau) P_l(\mu)$$

Substituting the above formula into RT equation and using the orthogonality relation of the Legendre function,

$$\frac{d}{d\tau} \begin{bmatrix} I_0(\tau) \\ I_1(\tau) \\ I_2(\tau) \\ I_3(\tau) \end{bmatrix} = \begin{bmatrix} 0 & a_1 & 0 & -\frac{2}{3}a_2 \\ a_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3}a_3 \\ -\frac{2}{3}a_0 & 0 & -\frac{1}{3}a_2 & 0 \end{bmatrix} \begin{bmatrix} I_0(\tau) \\ I_1(\tau) \\ I_2(\tau) \\ I_3(\tau) \end{bmatrix} + \begin{bmatrix} \frac{2}{3}b_3 - b_1 \\ -b_0 \\ -\frac{1}{3}b_3 \\ \frac{2}{3}b_0 - \frac{1}{3}b_2 \end{bmatrix} e^{-\tau/\mu_0}$$



Cloud radiative transfer for solar radiation

According to the orthogonality of $P_1(\mu)$ and $P_3(\mu)$ in $0 \leq \mu \leq 1$ or $-1 \leq \mu \leq 0$, we can get

$$F^-(\tau) = 2\pi \int_0^{-1} I(\tau, \mu) P_1(\mu) d\mu = 2\pi \left[\frac{1}{2} I_0(\tau) - I_1(\tau) + \frac{5}{8} I_2(\tau) \right]$$

$$\mathcal{F}^-(\tau) = 2\pi \int_0^{-1} I(\tau, \mu) P_3(\mu) d\mu = 2\pi \left[-\frac{1}{8} I_0(\tau) + \frac{5}{8} I_2(\tau) - I_3(\tau) \right]$$

$$F^+(\tau) = 2\pi \int_0^1 I(\tau, \mu) P_1(\mu) d\mu = 2\pi \left[\frac{1}{2} I_0(\tau) + I_1(\tau) + \frac{5}{8} I_2(\tau) \right]$$

$$\mathcal{F}^+(\tau) = 2\pi \int_0^1 I(\tau, \mu) P_3(\mu) d\mu = 2\pi \left[-\frac{1}{8} I_0(\tau) + \frac{5}{8} I_2(\tau) + I_3(\tau) \right]$$

Therefore, we can get the solution

$$\begin{bmatrix} F^-(\tau) \\ \mathcal{F}^-(\tau) \\ F^+(\tau) \\ \mathcal{F}^+(\tau) \end{bmatrix} = \begin{bmatrix} \varphi_1^- e_1 & \varphi_1^+ e_3 & \varphi_2^- e_2 & \varphi_2^+ e_4 \\ \Phi_1^- e_1 & \Phi_1^+ e_3 & \Phi_2^- e_2 & \Phi_2^+ e_4 \\ \varphi_1^+ e_1 & \varphi_1^- e_3 & \varphi_2^+ e_2 & \varphi_2^- e_4 \\ \Phi_1^+ e_1 & \Phi_1^- e_3 & \Phi_2^+ e_2 & \Phi_2^- e_4 \end{bmatrix} \mathbf{G} + \begin{bmatrix} Z_1^- \\ Z_2^- \\ Z_1^+ \\ Z_2^+ \end{bmatrix} \exp(-f_0 \tau)$$



Cloud radiative transfer for solar radiation

a) two layer

The four principles of invariance governing the reflection and transmission of light beam

$$U(\mu, \mu_0) = R_2(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + 2 \int_0^1 R_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$

4DDA

$$D(\mu, \mu_0) = T_1(\mu, \mu_0) + 2 \int_0^1 R_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$R_{1,2}(\mu, \mu_0) = R_1(\mu, \mu_0) + U(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu}\right) + 2 \int_0^1 T_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$T_{1,2}(\mu, \mu_0) = T_2(\mu, \mu_0) \exp\left(-\frac{\tau_1}{\mu_0}\right) + D(\mu, \mu_0) \exp\left(-\frac{\tau_2}{\mu}\right) + 2 \int_0^1 T_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$

$$U(\mu, \mu_0) = R_2(\mu, \mu_0) e^{-\tau_1/\mu_0} + 2 \int_0^1 R_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$

4SDA

$$D(\mu, \mu_0) = T_1(\mu, \mu_0) + 2 \int_0^1 R_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$R_{1,2}(\mu, \mu_0) = R_1(\mu, \mu_0) + 2 \int_0^1 \tilde{T}_1^*(\mu, \mu') U(\mu', \mu_0) \mu' d\mu'$$

$$\int_0^1 P_1(\mu) P_3(\mu) d\mu = 0$$

orthogonality

$$T_{1,2}(\mu, \mu_0) = T_2(\mu, \mu_0) e^{-\tau_1/\mu_0} + 2 \int_0^1 \tilde{T}_2(\mu, \mu') D(\mu', \mu_0) \mu' d\mu'$$



Cloud radiative transfer for solar radiation

We multiple $2P_1(u)du$ in both sides of the above equation and do integration from 0 to 1. Also we multiple $2P_3(u)du$ in both sides of the above equation and do integration from 0 to 1. Then, we can obtain

$$\mathbf{U}(\mu_0) = \mathbf{R}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{R}}_2\mathbf{D}(\mu_0)$$

$$\mathbf{D}(\mu_0) = \mathbf{T}_1(\mu_0) + \overline{\mathcal{R}}_1^*\mathbf{U}(\mu_0)$$

$$\mathbf{R}_{1,2}(\mu_0) = \mathbf{R}_1(\mu_0) + \overline{\mathcal{T}}_1^*\mathbf{U}(\mu_0)$$

$$\mathbf{T}_{1,2}(\mu_0) = \mathbf{T}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{T}}_2\mathbf{D}(\mu_0)$$

We get the solution

$$\mathbf{U}(\mu_0) = (\mathbf{E} - \overline{\mathcal{R}}_2\overline{\mathcal{R}}_1^*)^{-1}[\mathbf{R}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{R}}_2\mathbf{T}_1(\mu_0)]$$

$$\mathbf{D}(\mu_0) = \mathbf{T}_1(\mu_0) + \overline{\mathcal{R}}_1^*(\mathbf{E} - \overline{\mathcal{R}}_2\overline{\mathcal{R}}_1^*)^{-1} \times [\mathbf{R}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{R}}_2\mathbf{T}_1(\mu_0)]$$

$$\mathbf{R}_{1,2}(\mu_0) = \mathbf{R}_1(\mu_0) + \overline{\mathcal{T}}_1^*(\mathbf{E} - \overline{\mathcal{R}}_2\overline{\mathcal{R}}_1^*)^{-1} \times [\mathbf{R}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{R}}_2\mathbf{T}_1(\mu_0)]$$

$$\mathbf{T}_{1,2}(\mu_0) = \mathbf{T}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{T}}_2\mathbf{T}_1(\mu_0) + \overline{\mathcal{T}}_2\overline{\mathcal{R}}_1^*(\mathbf{E} - \overline{\mathcal{R}}_2\overline{\mathcal{R}}_1^*)^{-1} \times [\mathbf{R}_2(\mu_0)e^{-\tau_1/\mu_0} + \overline{\mathcal{R}}_2\mathbf{T}_1(\mu_0)]$$



Cloud radiative transfer for solar radiation

b) multi-layer

$$\begin{aligned}\mathbf{U}_{k+1}(\mu_0) &= [\mathbf{E} - \overline{\mathcal{R}}_{k+1,N} \overline{\mathcal{R}}_{1,k}^*]^{-1} \\ &\quad \times \left[\mathbf{R}_{k+1,N}(\mu_0) \exp\left(-\frac{\tau_{1,k}}{\mu_0}\right) + \overline{\mathcal{R}}_{k+1,N} \mathbf{T}_{1,k}(\mu_0) \right] \\ \mathbf{D}_{k+1}(\mu_0) &= \mathbf{T}_{1,k}(\mu_0) + \overline{\mathcal{R}}_{1,k}^* [\mathbf{E} - \overline{\mathcal{R}}_{k+1,N} \overline{\mathcal{R}}_{1,k}^*]^{-1} \\ &\quad \times \left[\mathbf{R}_{k+1,N}(\mu_0) \exp\left(-\frac{\tau_{1,k}}{\mu_0}\right) + \overline{\mathcal{R}}_{k+1,N} \mathbf{T}_{1,k}(\mu_0) \right]\end{aligned}$$

Then, the flux is

$$\begin{aligned}F_{k+1}^\uparrow &= \mu_0 F_0 \boldsymbol{\mu} \cdot \mathbf{U}_{k+1}(\mu_0) \\ F_{k+1}^\downarrow &= \mu_0 F_0 \boldsymbol{\mu} \cdot \mathbf{D}_{k+1}(\mu_0) + \mu_0 F_0 \exp\left(-\frac{\tau_{1,k}}{\mu_0}\right)\end{aligned}$$

Zhang et al J. Atmos. Sci., 2013b

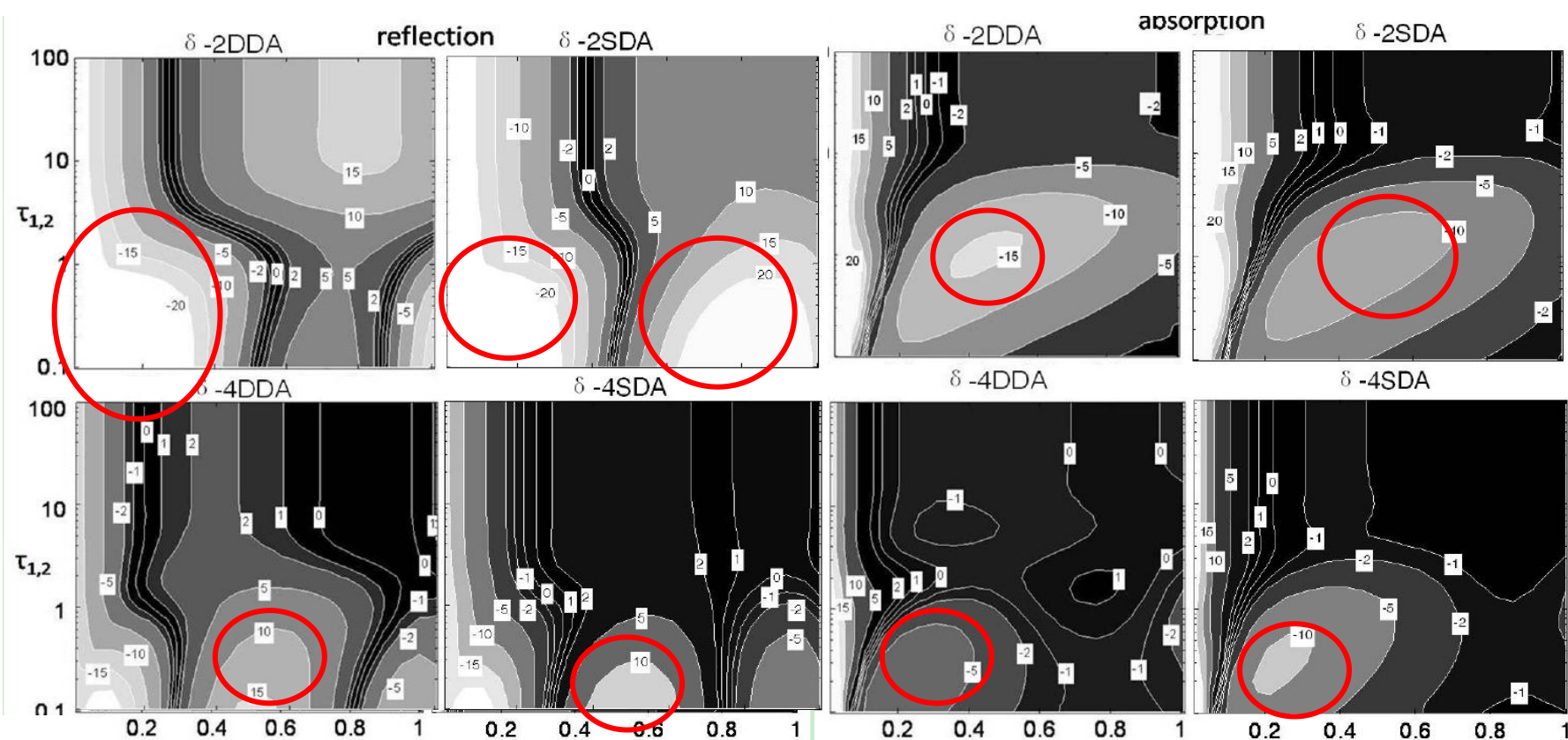


Cloud radiative transfer for solar radiation

Comparison results and discussion

a. Double layer

$$\omega_1 = \omega_2 = 0.9 \quad g_1 = 0.837 \text{ and } g_2 = 0.861$$

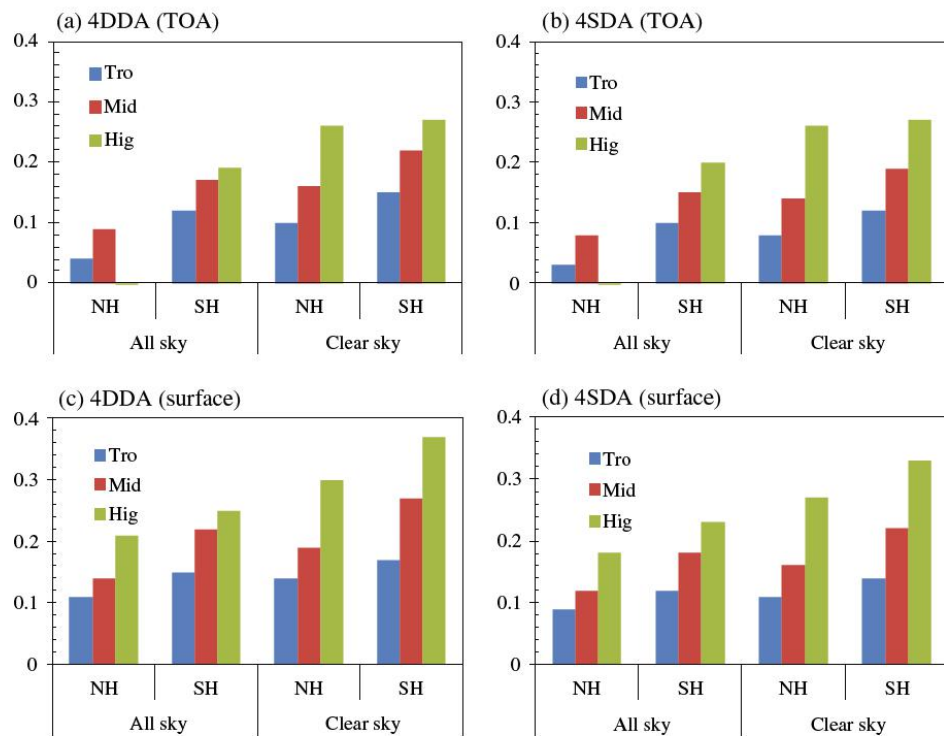


Atmospheric General Circulation Model of the Beijing Climate Center (BCC_AGCM2.0.1)

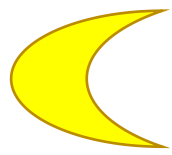
Table 1. Global annual means of simulated total aerosol shortwave DREs for the year 2000 (unit: $W m^{-2}$).

	All sky			Clear sky		
	TOA	ATM	SFC	TOA	ATM	SFC
Base	-2.01	2.19	-4.20	-4.49	2.21	-6.69
δ -4DDA	-2.21 (10%)	2.59 (18%)	-4.80 (14%)	-5.21 (16%)	2.74 (24%)	-7.95 (19%)
δ -4SDA	-2.17 (8%)	2.51 (15%)	-4.68 (12%)	-5.10 (14%)	2.61 (18%)	-7.71 (15%)

TOA, ATM, and SFC represent top of the atmosphere, atmosphere, and surface, respectively. Values in parentheses are relative differences in aerosol shortwave DREs between two- and four-stream algorithms.



Zhang Hua, Zhili Wang*,
Feng Zhang, Xianwen Jing,
2015, International
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Cloud radiative transfer for Infrared radiation

Introduction

- Solving the radiative transfer equation (RTE) is a key issue when dealing with radiative processes. For infrared radiation, an **absorption approximation (AA)** is used in most current climate models ([Oreopoulos et al. 2012](#)).
- Studies have shown that **AA leads to larger errors in cloudy sky cases**. Conversely, if scattering is considered, we need to use the **discrete-ordinates method (DOM)** and the calculation process is complicated.
- The aim of this study was to **develop 4DDA and VIM to establish a more efficient infrared radiative transfer method for a scattering medium**, whose accuracy would be comparable to that of DOM.

Cloud radiative transfer for Infrared radiation

Infrared radiative transfer equation

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - (1 - \omega) B(\tau) - \frac{\omega}{2} \int_{-1}^1 I(\tau, \mu') P(\mu, \mu') d\mu'$$

cosine of zentih angle **radiative intensity** **the Planck function at optical depth τ** **single-scattering albedo** **azimuthal independent phase function**



It's an integro-differential equation and it has no exact solution.



Cloud radiative transfer for Infrared radiation

1. Absorption Approximation (AA)

In absorption approximation (AA), **the scattering phase function P is simplified as a δ function**, and the infrared RTE becomes

$$\mu \frac{dI(\tau, \mu)}{d\tau} = (1 - \omega)I(\tau, \mu) - (1 - \omega)B(\tau)$$

So the upward intensity at $\tau=0$ is

$$I(0, \mu) = I(\tau_0, \mu)e^{-(1-\omega)\tau_0/\mu} + \frac{1-\omega}{\mu\beta+1-\omega} [B_0 - B_1 e^{-(1-\omega)\tau_0/\mu}]$$

And the downward intensity at $\tau=\tau_1$ is

$$I(\tau_1, -\mu) = I(0, -\mu)e^{-(1-\omega)\tau_1/\mu} + \frac{1-\omega}{\mu\beta-1+\omega} [B_0 e^{-(1-\omega)\tau_1/\mu} - B_1]$$



Cloud radiative transfer for Infrared radiation

Variational Iteration Method (VIM)

For a general nonlinear system:

$$\underbrace{L v(x)}_{\text{a linear term}} + \underbrace{N v(x)}_{\text{a nonlinear term}} = \underbrace{h(x)}_{\text{an inhomogeneous term}}$$

If $v(x)$ is found in n -th iteration, the $(n+1)$ -th-order solution is

$$v^{n+1}(x) = v^n(x) + \int_0^x \underbrace{\lambda(\zeta)}_{\text{a restricted variation, which equals to 0 after variation}} [L v^n(\zeta) + N \underbrace{\tilde{v}^n(\zeta)}_{\text{a general Lagrange multiplier, which can be identified by using variational theory}} - h(\zeta)] d\zeta$$

Cloud radiative transfer for Infrared radiation

For the infrared RTE

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - (1 - \omega)B(\tau) - \frac{\omega}{2} \int_{-1}^1 I(\tau, \mu') P(\mu, \mu') d\mu'$$

The functional reiteration can be deduced as

$$I^{n+1}(s, \pm\mu) = I^n(s, \pm\mu) + \int_{\tau_s}^{\tau} \lambda^{\pm}(s) \left[\frac{dI^n(s, \pm\mu)}{ds} \mp \frac{I^n(s, \pm\mu)}{\mu} \pm \frac{1 - \omega}{\mu} B(s) \right. \\ \left. \pm \frac{\omega}{2\mu} \int_{-1}^1 \tilde{I}^n(s, \mu') P(\pm\mu, \mu') d\mu' \right] ds$$

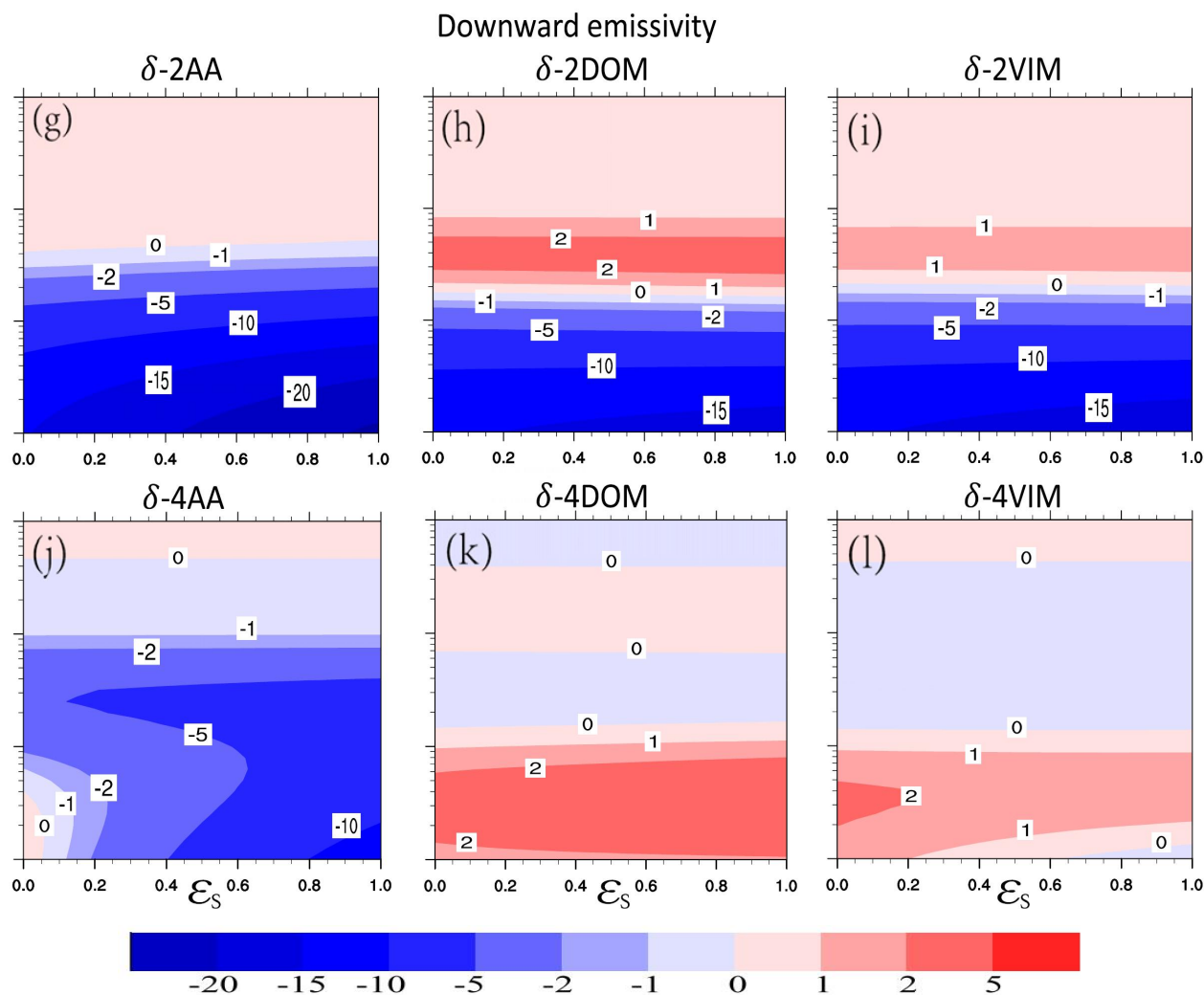
The AA expression can be used as the initial 0th-order solution, and the scattering effect is included in the first-order solution



Cloud radiative transfer for Infrared radiation

a. Single Medium

$$\omega=0.7105, g=0.9044$$



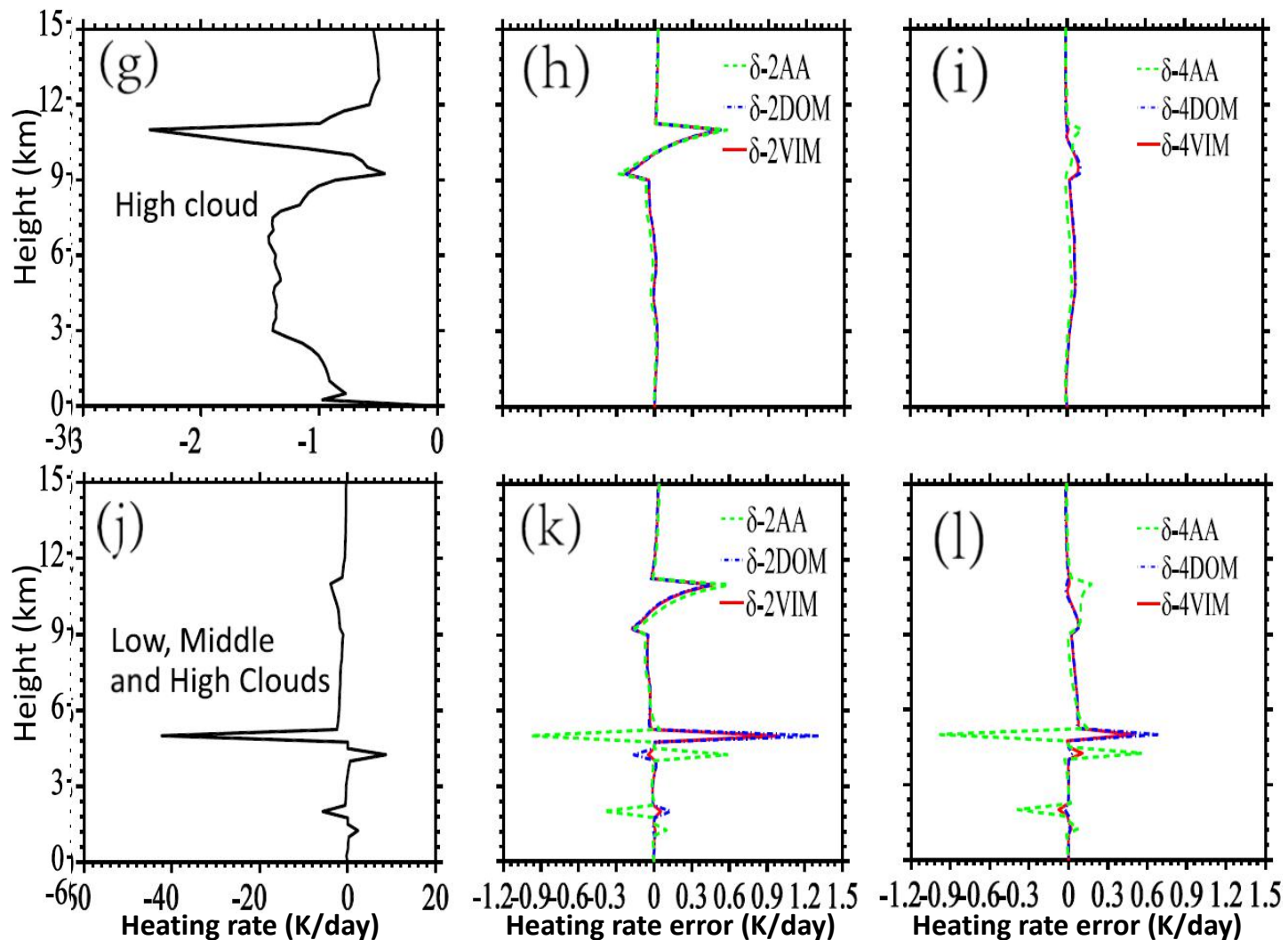
δ -2VIM is similar to δ -2DOM in the small optical depth but more accurate than δ -2DOM in the large optical depth.

δ -4VIM and δ -4DOM are much more accurate than δ -4AA.



Cloud radiative transfer for Infrared radiation

b. Multilayer atmosphere





Cloud radiative transfer for Infrared radiation

Efficiency

Computing times of various infrared radiative transfer methods (normalized by the computing time of δ -2AA)

	δ -2AA	δ -2DOM	δ -2VIM	δ -4AA	δ -4DOM	δ -4VIM
Algorithm only	1.00	2.22	2.04	1.44	14.82	5.36
Radiation model	1.00	1.41	1.38	1.11	5.90	2.27

δ -2VIM is slightly faster
than δ -2DOM

δ -4VIM is more than
twice as fast as δ -4DOM

Zhang et al. J. Atmos. Sci., 2017

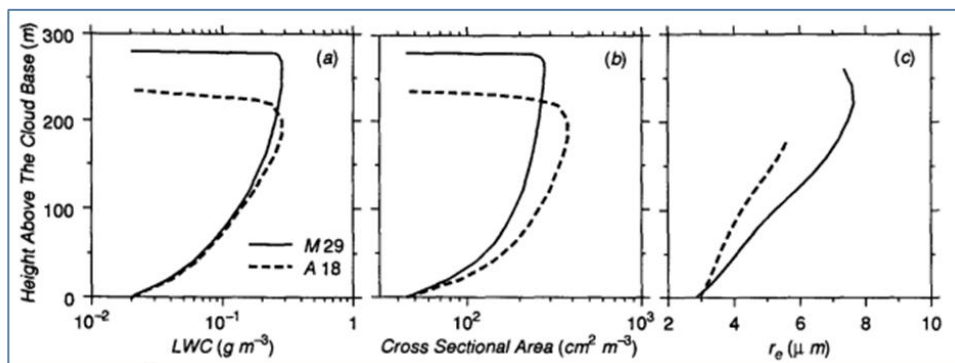
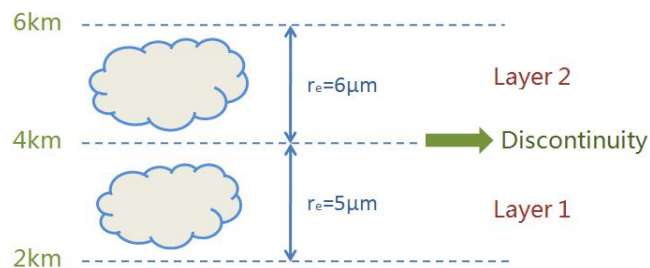


Vertically internally inhomogeneous Cloud

- 已有大量观测表明，云的微物理结构随距离云底的高度增加呈较大的变化。对于典型的对流云而言，其云水含量在云底分布最少，随着距离云底高度的增加而增加，在接近云顶处达到最大；而通常在对流云上层观测到的云滴大小比云的下层大。

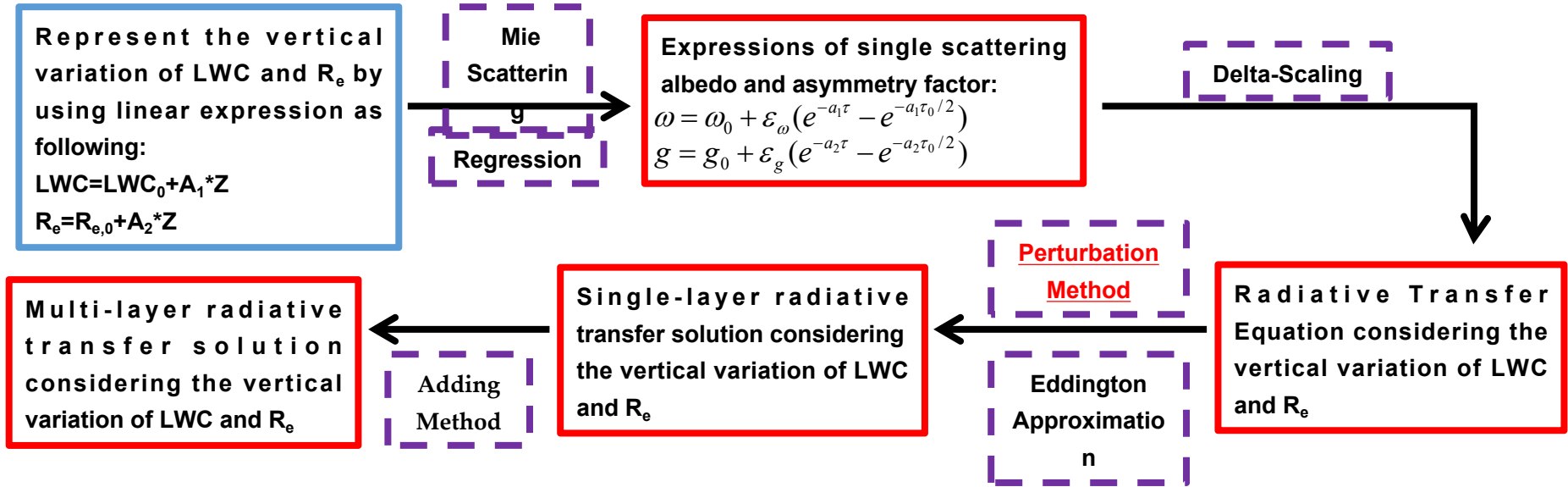
- 现有模式中将网格尺度的云微物理特性进行不连续分层的做法破坏了云微物理特性变化的连续性，从而导致模式层交界处的云微物理特性存在突变。

Li et al., 1994;
Duan et al., 2010





Vertically internally inhomogeneous Cloud



$$F^+ = F_0^+ + \varepsilon_\omega F_1^+ + \varepsilon_g F_2^+,$$

$$F^- = F_0^- + \varepsilon_\omega F_1^- + \varepsilon_g F_2^-.$$

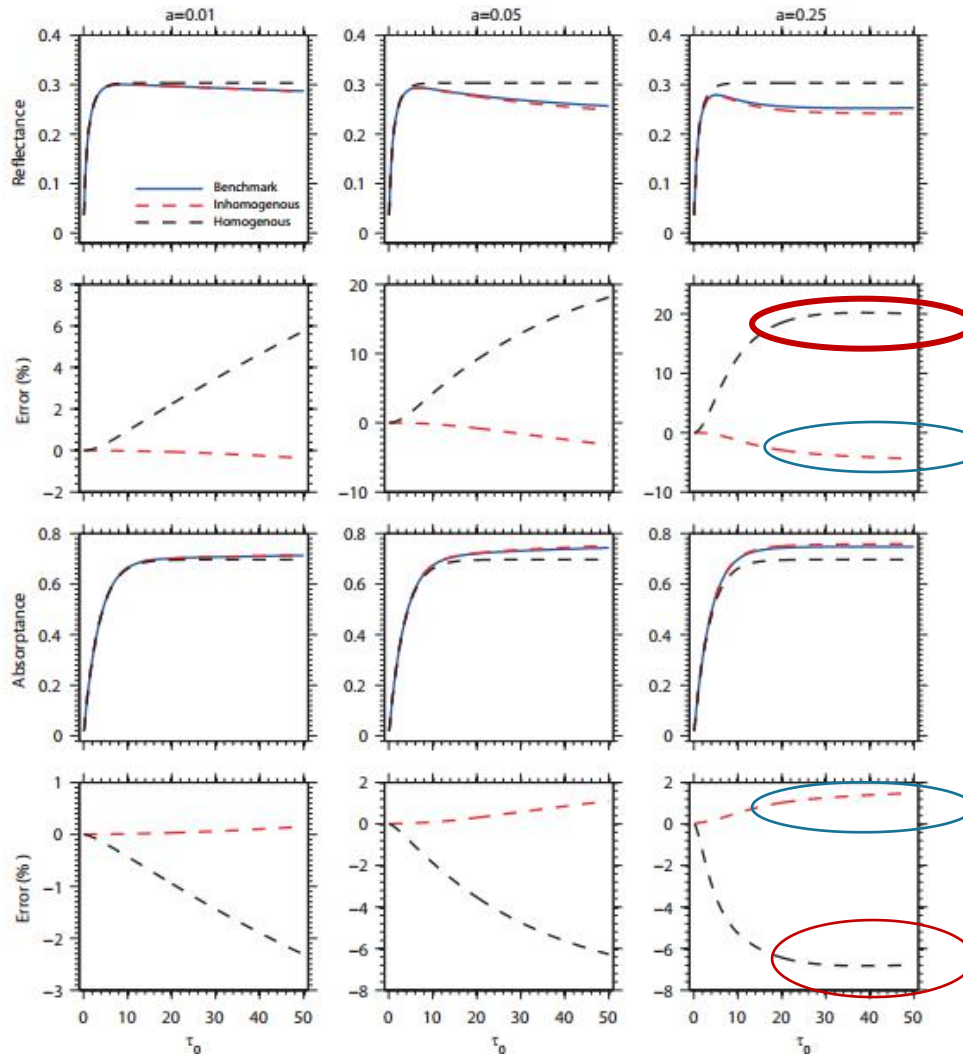
$$\frac{dF^+}{d\tau} = \left[\gamma_1^0 + \gamma_1^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_1^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}) \right] (F_0^+ + F_1^+ \varepsilon_\omega + F_2^+ \varepsilon_g)$$

$$- \left[\gamma_2^0 + \gamma_2^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_2^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}) \right] (F_0^- + F_1^- \varepsilon_\omega + F_2^- \varepsilon_g)$$

$$- \left[\hat{\omega} \gamma_3^0 + \gamma_3^0 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \hat{\omega} \gamma_3^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}) \right] F_s e^{-\tau/\mu_0}$$

Vertically internally inhomogeneous Cloud

1. Idealized medium



homogeneous solutions

inhomogeneous solutions

$$g(\tau) = 0.75$$

$$\omega(\tau) = 0.9 - 0.05(e^{-a\tau} - e^{-a\tau_0/2}).$$

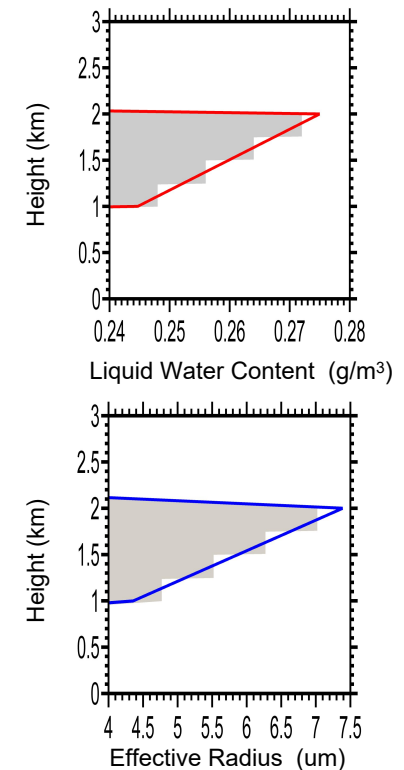
Zhang et al. J. Atmos. Sci., 2018

Vertically internally inhomogeneous Cloud

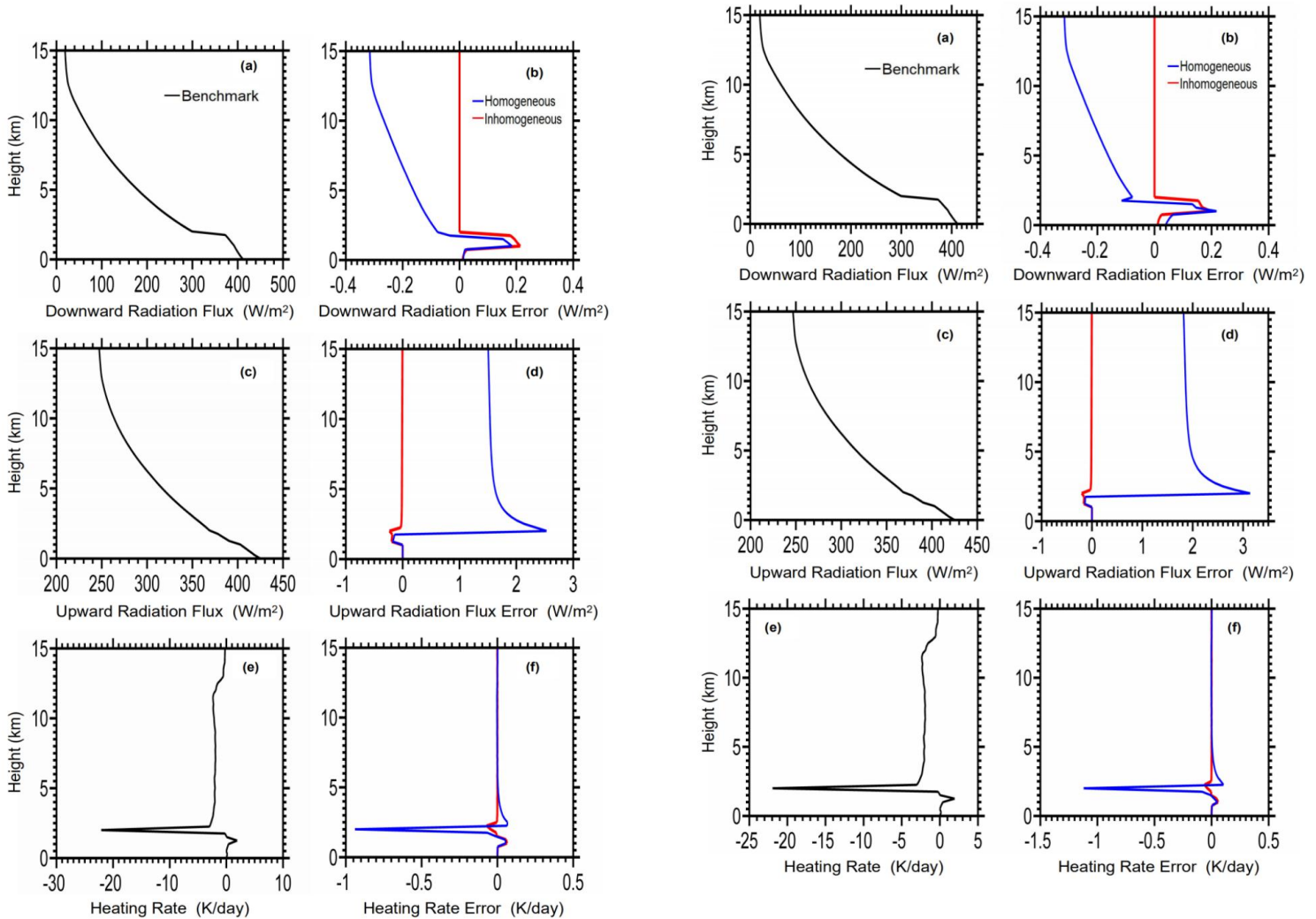
The accuracy and efficiency of the new radiative transfer scheme :

■ Simulating Description

Name	Benchmark	Homogeneous	Inhomogeneous
Atmospheric Profile	The midlatitude winter atmosphere with a subdivision of 400 layers each of which having a geometrical thickness of 0.25 km.		
Cloud Location		1 - 2 km	
Vertical Resolution of Cloudy Area	100 layer	4 layer	4 layer
LWC within Cloud (g m⁻³)	$0.245 + 0.00003z$ where z varies 0 - 1000 m	0.248, 0.256, 0.264, 0.272	$0.245 + 0.00003z$ where z varies 0 - 1000 m
R_e within Cloud (um)	$4.39 + 0.003z$ where z varies 0 - 1000 m	4.77, 5.52, 6.27, 7.02	$4.39 + 0.003z$ where z varies 0 - 1000 m



Vertically internally inhomogeneous Cloud

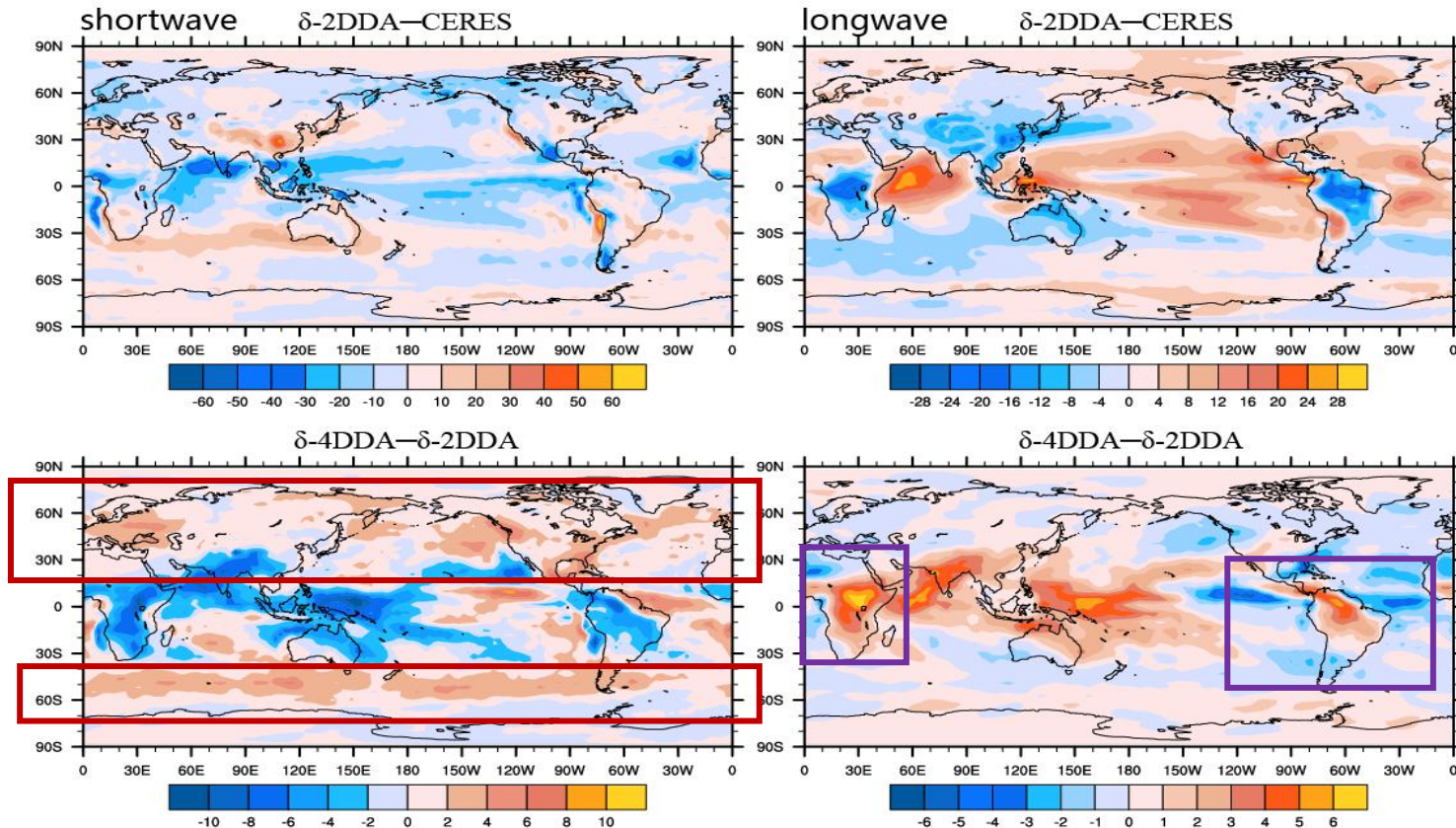


Shi, Zhang*al Optics
Express. 2019



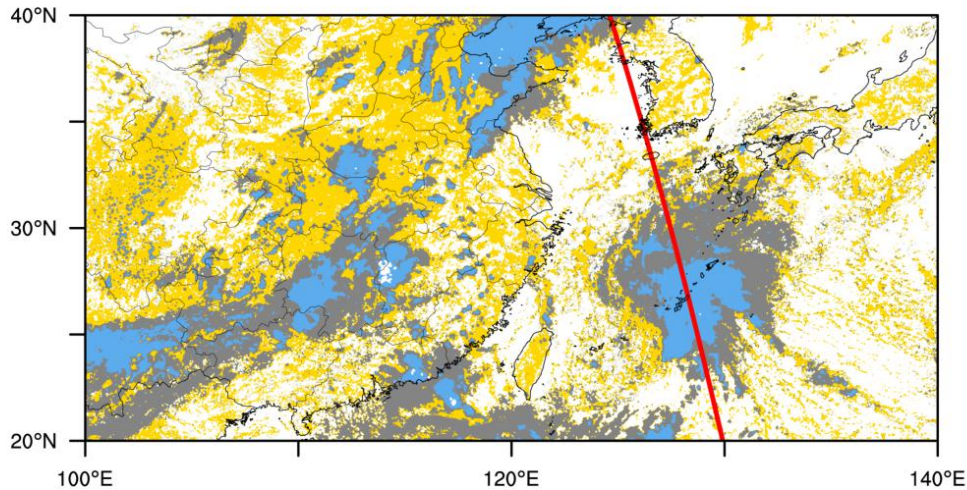
Impact of four-stream adding method on cloud radiative effects in climate model

- BCC_AGCM2.0.1
- Run Time: 1975-2010, last 30 years for analysis



DCS identification

(a) Cloud type defined by AHI cloud product (2016-08-07 04:50)



(b) Cloud type defined by AHI cloud product



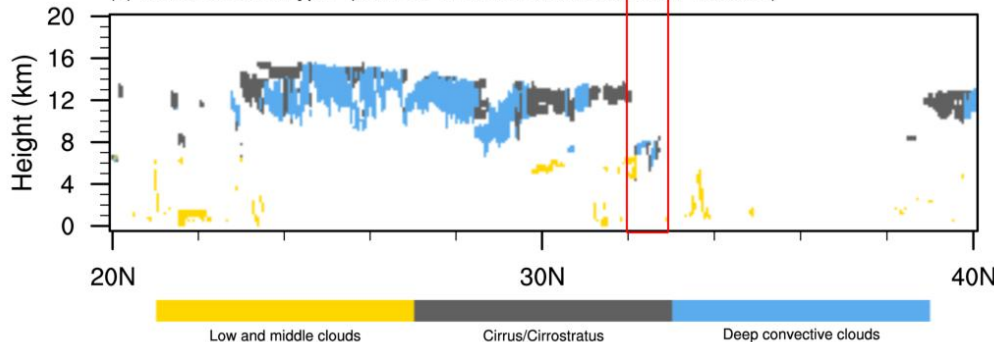
(c) Cloud type defined by split window algorithm



(d) Cloud type defined by single-band BT threshold



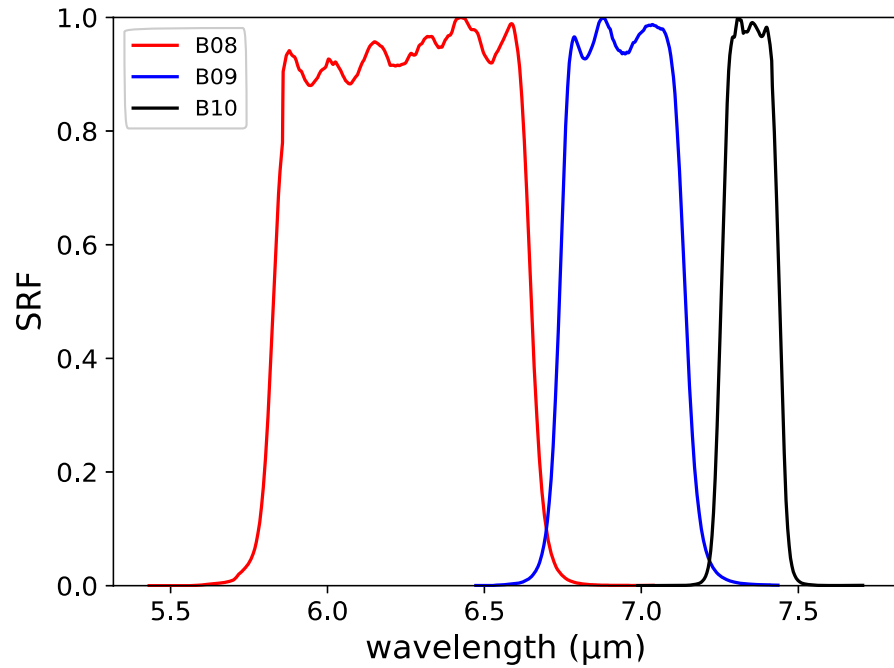
(e) CALIPSO cloud type (2016-08-07 04:52:42 to 2016-08-07 04:58:18)



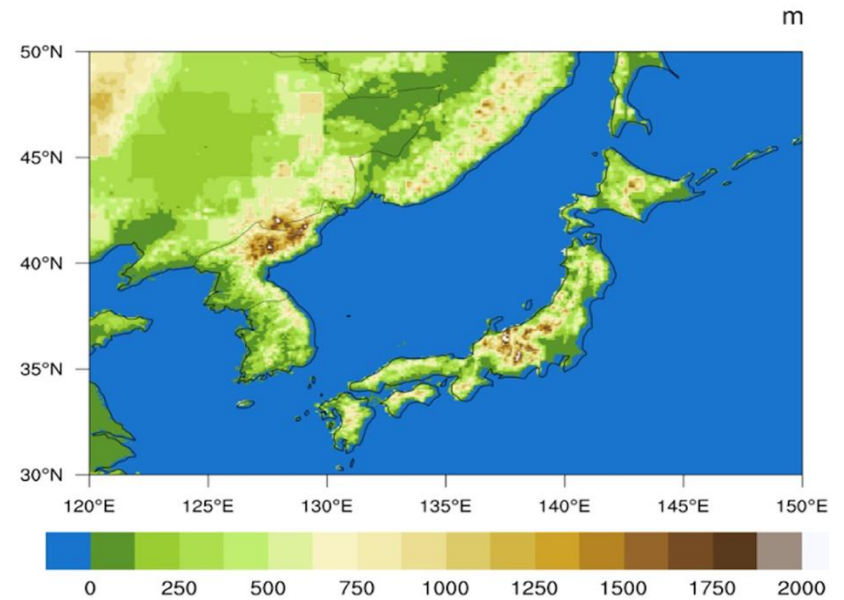
Cloud type map from AHI cloud product based on ISCCP at 04:50 UTC on 7 August 2016 over region (20°N-40°N, 100°W-140°W) with CALIPSO orbit indicated by the red line.

Li, Zhang* et al.,
Submitted to Climate Dynamics

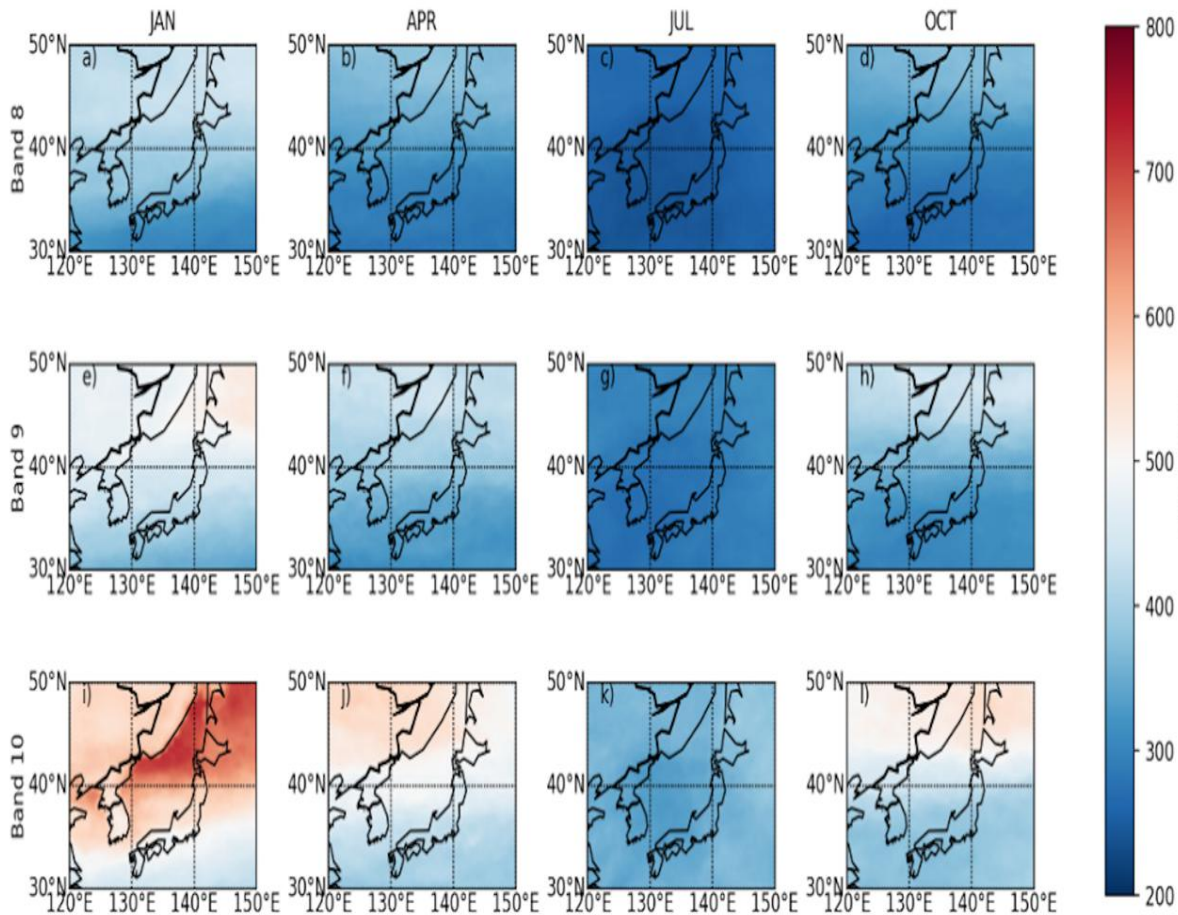
Best Water Vapor Information Layer



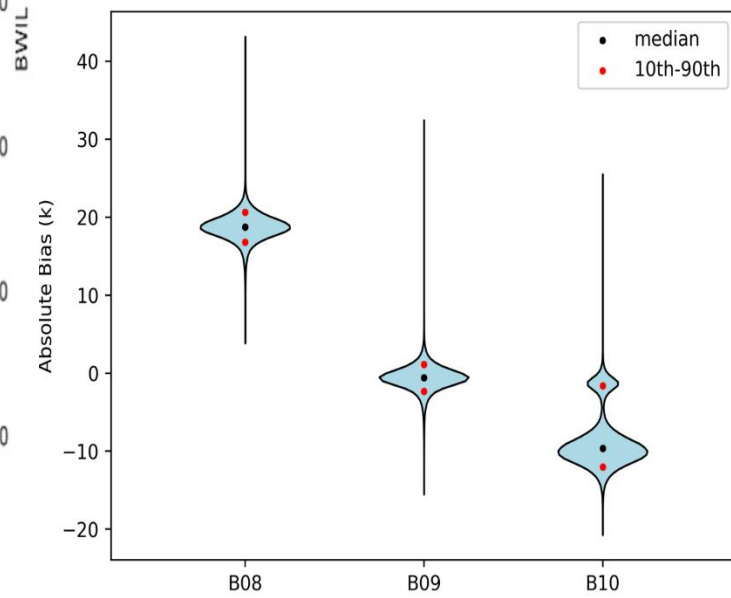
SRFs of the water vapor absorption IR bands for AHI.



Elevation distribution over the typical region of East Asia. Units are m.



The mean height of BWIL over the typical region of East Asia in January, April, July and October



The violin plot of absolute biases between the AHI observations and simulated brightness temperatures for Band 8, Band 9 and Band 10.



THANKS!

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Homepage: <https://atmsci.fudan.edu.cn/60/f3/c14809a221427/page.htm>

